Using Computer Algebra System (CAS) Software to Address Physics Education Problems

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Abstract

Early preliminary results of use of a particular CAS (Mathematica) in the introductory physics courses at the University of Houston - Downtown (UHD) are reported. The courses addressed are algebra- and trigonometry-based lecture and laboratory courses, with the greatest emphasis on the laboratory courses. The program was first implemented in January of 2009, at the beginning of the Spring Semester. The initial goals of the program are four-fold: (1) to emphasize to students the utility of symbols in the solution of physics problems, (2) to avoid calculation errors in numerically evaluating expressions that students find complex or tedious, (3) to provide students some familiarity with CAS software, and (4) to display the ability of CAS software to generate correct and understandable symbolic solutions to physics problems. The use by students of the CAS is optional. They are provided no formal training in use of the software, but rather "on-the-job" training by instructors through guiding examples of software use supplied in the laboratory manual and in posted problem solutions in selected lecture course sections.

Introduction

The University of Houston - Downtown (UHD) is a metropolitan open-admissions commuter university with a student census of about 11,500. The physics teaching program is housed in the Department of Natural Sciences, which does not offer a bachelor's degree in physics. The program's primary academic purpose is the service offering of algebra- and trigonometry-based introductory physics in a two-semester series of lecture and laboratory courses. Currently, these service courses register approximately 180 students each semester, students who are primarily pursuing degrees in biology, engineering technology, chemistry and other professional or technology majors. The program currently offers no calculus-based introductory courses, and none of the traditional upper-level undergraduate courses in mechanics, electrodynamics, thermodynamics, mathematical methods or quantum mechanics.

As a member institution of the University of Houston System (UHS), the university has access to a site license for a powerful example of CAS software, the Mathematica program offered by Wolfram Research, Inc. The terms of the site license allow free use of the software on personal computers by both faculty and students of UHD (and other institutions in the UHS).

The author is a long-time user of Mathematica, and has long been interested in the potential value of such software as an educational tool in undergraduate mathematics, physics, engineering and other scientific fields. Limited experiments in application of CAS software to undergraduate teaching are ongoing in a number of research institutions, where site licenses for such software are common.

Upon arriving at UHD in late summer, 2008, the author became aware of the institutional site license for Mathematica, and opened discussions with the department chair and other physics instructors on the topic of making use of this software in teaching the physics courses and laboratories. Consensus was reached that the mechanics (Physics I) laboratory course would be revamped for initial use with Mathematica in the Spring 2009 semester. The software was installed on computers at each of ten tables in the room used for this laboratory, and in the major computer laboratory used by the students on campus (about 180 computers) during December 2008 and January 2009.

The laboratory manual was re-written to accommodate use of Mathematica, and in addition to accommodate other changes being made in the laboratory course. The other changes involved a consistent emphasis on determinations of experimental uncertainties, and in a reduction of scope (fewer experiments) with greater depth of analysis of the smaller number of experiments. The modified Physics I laboratory was implemented in January 2009.

Educational Goals

The educational goals for instituting use of the CAS software are aligned with the general educational goals for the physics courses, both in their lecture and laboratory versions. These are specifically enumerated below.

★ Emphasis on Symbolic Solutions to Problems

Conveying the importance of mathematical symbols as essential tools in analysis is a major aspect of teaching throughout the undergraduate science and mathematics curriculum. This requires a large investment of teaching effort because students typically begin undergraduate study with a trained impulse to immediately convert any problem to arithmetic, and to avoid any higher mathematics as much as possible. Thus students must first unlearn a bad habit before they can make progress with new and more demanding material. This difficulty is especially acute for the mathematically naive students who typically enter our course.

★ Avoiding Calculation Errors in Evaluating Complex Expressions

Time and effort that students spend struggling to correctly calculate numerical results is time lost from understanding physics concepts and solutions.

★ Gaining Familiarity with CAS Software

Most of our students that are awarded degrees can expect in their work careers to become competent in the use of technical computing software of some sort. CAS software is the epitome of software of this type, even including financial and work-flow software. We presume that any familiarity gained of Mathematica will pay dividends to the students in their post-graduation work.

★ Gaining Understanding of CAS Generation of Symbolic Solutions to Problems

This is closely related to the first two goals. In dealing with real-world problems, even symbolic solutions can be difficult and tedious to perform, although such problems are avoided in introductory courses. An understanding of the capabilities of CAS software in dealing with such problems can pay significant dividends in technical career work.

Methods

Physical Changes

There were basically no changes made in the traditional laboratory experiments, aside from reducing their number, deepening the analysis, and making the software available on student computers in the laboratory room. A projector and screen were added to the room.

The number of experiments was reduced from ten to six, and two weeks (two laboratory class meetings) were allotted to each experiment.

Written Materials

All of the written materials needed by the students are placed on the Blackboard Vista (course management software) web site for the course. The basic texts are briefly described in the following list.

◇ Lab Manual

There is a lab manual file for each of the six (6) experiments performed, posted on the class web site in two forms: as a Mathematica file (which is actually used during laboratory class meetings), and as an Adobe Acrobat file (which students can download and read on any computer with the Acrobat Reader software installed).

◇ Lab Report Template

This file provides detailed instructions regarding material to be included in the lab report, and furthermore provides a standard template for student production of their own lab reports. The template is provided in three file formats: Microsoft Office Word 2007, Mathematica, and Adobe Acrobat. In the author's sections, students may electronically submit reports in either of the first two formats. The other two laboratory instructors require submission of printed reports.

◇ Experimental Uncertainties Appendix

This file, posted in the file formats of Mathematica and Adobe Acrobat, provides instruction on the meaning, use and determination of experimental uncertainties. This file describes statistical and other methods appropriate for determination of uncertainties in directly measured data and in quantities calculated from data. The statistical methods are simplified for direct applications like those encountered in the laboratory course.

◇ **Laboratory Discussion by the Instructor**

This aspect follows traditional methods, using the whiteboard, projector and computers in the laboratory room. The instructor provides a brief description of how data of the experiment are entered, and how the software can be used in aid of calculations for each experiment.

Results

★ **Emphasis on Symbolic Solutions to Problems**

Upon first approach to the software, students expect it to be a "super calculator". Indeed, it is a super calculator, but students quickly and directly learn that the software's very fast and effortless achievement of numerical wonders depends in a very direct way on the correct assignment of the numerical values of the application to symbols that are subsequently used in expressions to be numerically evaluated. The students are shown in a very explicit manner that the road to numerical wizardry is paved with intermediate symbolic assignments.

We are seeking ways to measure the effectiveness of the software in conveying this concept to students. Anecdotal evidence is quite positive; this evidence arises from observation of students analyzing their laboratory data, and discussing with them the process and the results they obtain. As a learning experience, it would appear to be considerably more effective than is, say, students' attentive observation of an instructor working through an example or problem in the classroom.

A more direct expression of the power of CAS software lies in purely symbolic calculations. We include in the lab manual examples of using the software to solve problems using only concepts expressed symbolically, such as kinematic equations and Newton's second law applications. These examples appear to be less effective than those in which symbols are assigned numerical values. The relatively smaller effect may be due to deficiencies in the choice or presentation of the examples, or it may simply reflect student unfamiliarity with connecting the symbolic solutions with the original problem, and with the idea that contemplating the meaning of the solutions is valuable.

★ **Avoiding Calculation Errors in Evaluating Complex Expressions**

The most common spontaneous expressions of appreciation by students of the CAS software occur in reference to this capability. Numerical evaluation of expressions complex enough that our students using hand-held calculators find them time-consuming, tedious, and prone to error are accomplished instantly by software, once the pertinent symbols have been assigned numerical values.

A striking example of this occurs in the projectile motion experiment. At the end of this experiment's analysis, the students must calculate the range of a projectile launched from a measured height at a measured oblique angle and measured release velocity. The calculation must include the uncertainty in the range. Using the software, students were able to quickly and easily obtain the numerical result, thereby seeing without agony the *essential point*: the mathematics produced a result in excellent agreement with their independent measurement of this range. In previous years of teaching this experiment, much less than ten percent of my students, using calculators, and including students in calculus-based courses, were able to correctly complete this calculation. And without a successful calculation of this result, the punch line of the experiment is greatly blunted.

To make this example more specific, the solution for the range in this case is

$$r_f = \frac{v_i^2 \, \mathrm{Sin}[2\,\theta]}{2\,g} \left(1 + \left(1 + \frac{2\,g\,h_e}{v_i^2 (\mathrm{Sin}[\theta])^2} \right)^{1/2} \right) \tag{1}$$

where v_i is the release velocity, θ the release angle and h_e the release height. And the experimentally determined calculation of the range, using in this case a "brute force" method for the uncertainty, is then

$$r_f = r_{f_b} \pm \Delta \mathrm{r} = \left(\frac{v_{ib}^2 \, \mathrm{Sin}[2\,\theta_b]}{2\,g} \left(1 + \left(1 + \frac{2\,g\,h_{eb}}{(\mathrm{Sin}[\theta_b])^2} \right)^{1/2} \right) \right) \pm \left(\mathrm{Max}\Big[\big(r_{f_{max}} - r_{f_b} \big), \big(r_{f_b} - r_{f_{min}} \big) \Big] \right) \tag{2}$$

where the experimentally determined quantities have the forms

$$h_e = h_{eb} \pm \Delta \mathrm{h}_e \tag{3}$$
$$v_i = v_{ib} \pm \Delta \mathrm{v}_i$$
$$\theta = \theta_b \pm \Delta\theta$$

Translating these into Mathematica code as used by the students results in

$$\text{rfb=} \frac{\text{vib}^2 \ \text{Sin}[2\theta b]}{2g} \left(1 + \left(1 + \frac{2g \ \text{heb}}{\text{vib}^2 \ (\text{Sin}[\theta b])^2}\right)^{1/2}\right);$$

$$\text{rfmax=} \frac{(\text{vib}+\Delta v)^2 \ \text{Sin}[2(\theta b+\Delta\theta)]}{2g} \left(1 + \left(1 + \frac{2g \ (\text{heb}+\Delta h)}{(\text{vib}-\Delta v)^2 \ (\text{Sin}[(\theta b-\Delta\theta)])^2}\right)^{1/2}\right);$$

$$\text{rfmin=} \frac{(\text{vib}-\Delta v)^2 \ \text{Sin}[2(\theta b-\Delta\theta)]}{2g} \left(1 + \left(1 + \frac{2g \ (\text{heb}-\Delta h)}{(\text{vib}+\Delta v)^2 \ (\text{Sin}[(\theta b+\Delta\theta)])^2}\right)^{1/2}\right);$$

`{rfb,Max[(rfmax-rfb),(rfb-rfmin)]}`

★ Gaining Familiarity with CAS Software

The value of this is emphasized in discussions by the instructor. However, we rely on the student's own initiative to gain this familiarity. We do not grade or evaluate in any way the proficiency gained by the student in use of the software. Student use of the software is entirely optional. Anecdotally, we find that a small fraction of the students become actively interested in learning how to use the software beyond the guided use during the laboratory class periods.

★ Gaining Understanding of CAS Generation of Symbolic Solutions to Problems

Again, this is not evaluated, since student use of the software is entirely optional. However, we are clearly not gaining much traction in this regard. The examples used in the lab manual and posted on the lecture web site, are largely not understood by the students, and not studied. The author hypotheses that this cannot improve without some structural changes in the lecture course, such as grading homework assignments, or permitting use of the software during tests.

Nonetheless, students are exposed to this capability of the software in the laboratory manual and during discussion of the analysis of the experimental data.

For specificity, we can return to the example previously used, the solution for the range of a projectile in free fall, repeated below in its most general form.

$$r_f = \frac{v_i^2 \ \text{Sin}[2\theta]}{2g} \left(1 \pm \left(1 + \frac{2 g \ h_e}{v_i^2 (\text{Sin}[\theta])^2}\right)^{1/2}\right) \qquad (4)$$

This solution arises from the kinematic displacement equation for constant acceleration

$$\vec{r} = \vec{r}_i + \vec{v}_i \ t + \frac{1}{2} \ \vec{a} \ t^2 \qquad (5)$$

which, when applied to the experiment becomes

$$\{r_f, 0\} = \{0, h_e\} + v_i \ t \ \{\text{Cos}[\theta], \text{Sin}[\theta]\} + \frac{1}{2} \ t^2 \ \{0, -g\} \qquad (6)$$

The code provided students for the solution is

```
{r,g,vi,tf,h,θ}=.;
Solve[r{1,0}==h{0,1}+vi tf{Cos[θ],Sin[θ]}+ 1/2 g tf²{0,-1},r,tf]
TrigReduce[%]
```

Execution of the cell above produces first a solution provided by the Solve command, and next a step of simplification provided by the TrigReduce command. A final step of simplification to the forms of (4) or (1) is left to the student.

★ Changes Noted in Student Lab Reports

The criteria for the lab reports changed significantly from those of previous semesters, due to other changes aside from use of Mathematica. These included an increased emphasis on experimental uncertainties and grading of both first draft and final versions of the reports. Here we attempt to limit our discussion to changes apparently due to student use of the software.

First, the number of errors in calculation has been reduced. This effect alone is positive, in that the conclusions reached by students rest on a firmer basis. This in turn appears to have lead some students to show a gain in appreciation of the validity of the mathematical models.

Our consistent attention to experimental uncertainties poses additional difficulties to students in both concept and calculation. The use of the CAS software largely obviates the calculation difficulties, so that students generally attain good results for the values of uncertainty in experimentally determined quantities. (They still display conceptual misunderstandings of the meaning of the uncertainties.)

Most of the students are using the graphics of the lab manual in their reports, and some of these are experimenting with modifying the graphics, though not always using Mathematica to perform the modifications.

About 10% of the students in the author's sections are submitting reports as Mathematica files, in which they can directly take advantage of the computing power of the software.

★ Initial Student Survey Responses

Students in the 2 lecture sections and the 3 laboratory sections taught by the author were surveyed by email. The students had 3 week days and a weekend in which to respond, and were offered extra credit grading for a complete response. There were 27 responses, approximately 33% of the students. The results are presented in Tables 1 and 2 below. Here the brand name of the particular software used at UHD has been replaced with the generic term "CAS software".

Table 1: Student responses to questions regarding their use of the CAS software. The number of respondents was 27.

Question		'Yes' Response (%)
1.	Do you use CAS software in the laboratory class meetings?	88.89
2.	Do you use CAS software in the computer lab in room S800?	77.78
3.	Have you downloaded CAS software to your own computer?	22.22
4.	Do you use CAS software on your own computer?	18.52
5.	Have you used any of the CAS software tutorials?	22.22

Table 2: Student responses to questions regarding their preferences for the CAS software. The number of respondents was 27.

Question		Score (1 to 5)
6.	I find CAS software to be helpful in my preparation of lab reports.	4.19
7.	I find CAS software to be helpful in my work on homework problems.	2.63
8.	I find CAS software to be helpful in my understanding of the mathematics involved in physics.	3.3
9.	I find access to CAS software to be helpful in the laboratory course.	4.19
10.	I find access to CAS software to be helpful in the lecture class.	2.59
11.	I prefer using CAS software to using a calculator for numerical calculations.	3.33
12.	I prefer using CAS software to using Excel for numerical calculations.	3.56
13.	I use CAS software not just for numerical calculations, but also to do algebra, plot graphs, or make graphics.	2.59
14.	The tutorials that come with CAS software are helpful to me in learning to use the software.	2.58
15.	I would like to have access to instruction on how to use CAS software outside the specific uses in the physics lecture or lab classes.	3.81

A large majority of the respondents use the software in the laboratory class, where typically data are entered and some results calculated, and almost as many continue this use in the student computer lab. But only about 1 in 5 respondents use the software on their own computers, where they might engage in greater exploration of its features.

They strongly agree that the software is helpful in preparation of their lab reports, but generally are not using the software in their lecture classes, nor do they use it for calculations outside those needed for the laboratory course work.

Conclusions

Conclusions at this point are assuredly premature, as the program is only six weeks old. Our goals are likely to change as our experience increases, and as we collect more data on the effectiveness of the program. The author can only speak for himself in saying that the experience has been uniformly positive.

Mathematica Work