A Prioritization Methodology For Orbital Debris Removal

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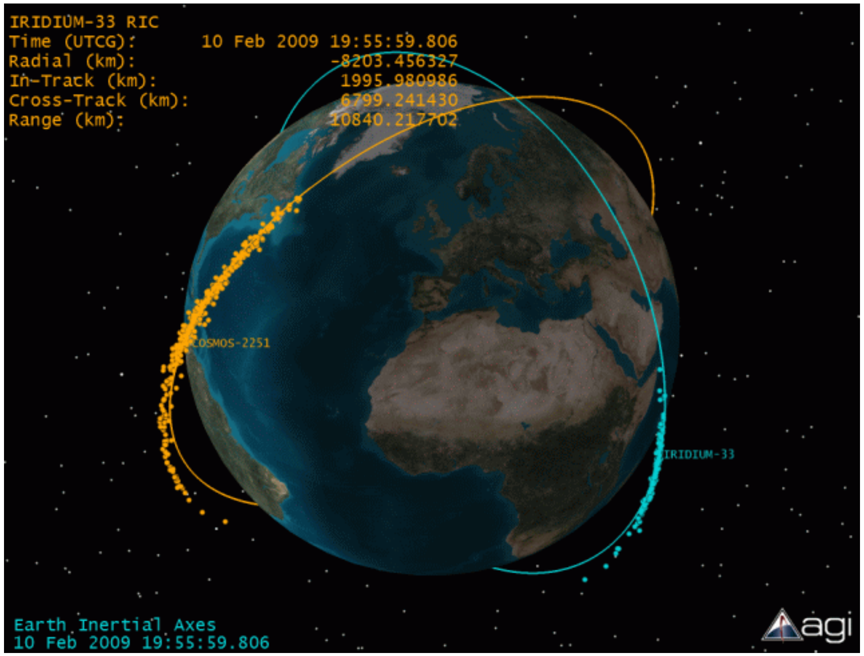
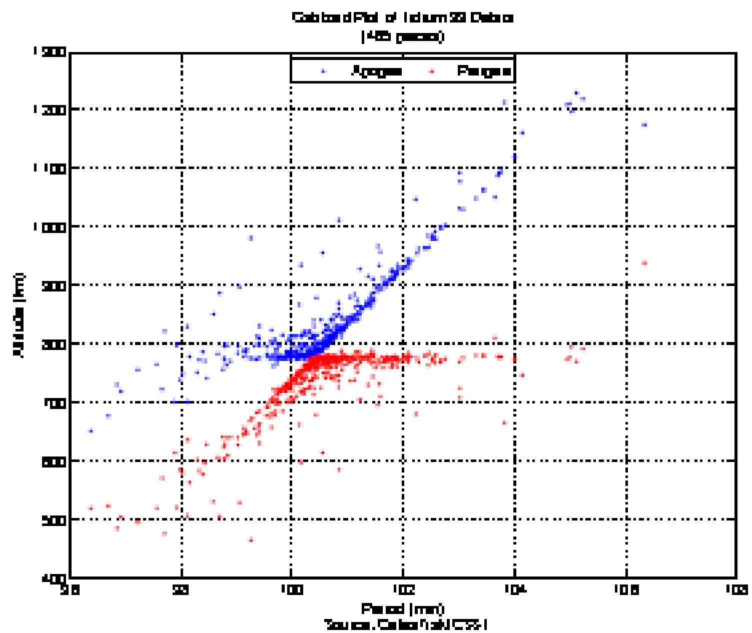
The Problem – On-Orbit Collisions …





Graphics – courtesy AGI --
http://www.celestrak.com/events/collision/

The Objective of This Work …

OBJECTIVE: To develop a **Prioritization Methodology for Orbital Debris Removal** based on the collisional rate equations presented by Talent (1992) in the context of a simple Particle-In-a-Box (PIB) debris environment evolution model. The discussion in the following slides will develop as follows …

- Review of the structure of the single-particle-equivalent PIB model,
- Discussion of the requirements for environment stability against catastrophic collisional cascade,
- Case 1: Long term evolution illustrated -- without target area removal,
- A look at the current LEO population – objects large and small,
- Case 2: Long term evolution illustrated -- with target area removal,
- Presentation of a collision rate equation for objects of different sizes,
- Prioritizing debris objects for removal – discussed and illustrated.

The PIB Model – Described

In 1988, inspired in part by the earlier work of Kessler and Cour-Palais (1978), the author of this presentation set out to develop a debris environment model based on a simple differential equation taking an approach that was thermodynamic in character. Originally presented at an AIAA/NASA/DoD conference (1990), the PIB model was later (1992) published as a peer-reviewed paper.

Two key elements of the PIB approach are as follows …

(1) Treat the entire LEO environment as one equivalent box (having global average characteristics) in which all objects resident might move about, and

(2) Describe all objects in the LEO environment in terms of one equivalent particle whose characteristics are determined by the total number of objects, total cross-sectional area, and total mass on orbit at any given moment.

The PIB Model – N, The State Parameter

In developing a mathematical model of an evolving system, one must first choose a relevant parameter as the “state” quantity.

In the present development, the number of objects, **N**, resident in the LEO environment at any given time is selected.

The reason for this choice is that if an object can be seen, it can be counted – the number of objects on orbit is a direct observable subject, of course, to an appreciation of possible incompleteness, especially at higher altitudes and smaller sizes.

The PIB Model – The Basic Equation

The basic PIB differential equation describing the change in the LEO population with time is presented here as ...

$$\dot{N} = A + BN + CN^2 = dN / dt$$

The form of the equation follows from the assumptions that …

(1) Deposition reflects the rate, **A**, at which users of the LEO environment choose to populate it with new objects,

(2) Decay due to atmospheric drag and/or random removal may be represented as a finite probability per unit time, **B**, of the decay (or removal) of any given LEO object, and

(3) The production of fragments during collisions, **C**, between members of the population may be developed in a fashion similar to that for collisions between particles in a gas.

The PIB Model – A: The Deposition Coefficient

In the PIB model, the coefficient **A** is expressed as …

$$A = L\left[(P1)(D1) + (FE)(PE)(DE)\right] - REM$$

where …

L = launches per year, worldwide

P1 = average number of pieces per launch

D1 = fraction of P1 meeting membership conditions

FE = fraction of launches eventually resulting in on-orbit non-collisional fragmentation

PE = number of fragments produced per explosion

DE = fraction of PE meeting membership conditions

REM = number of objects removed per year by deliberate retrieval

The PIB Model – B: The Removal Coefficient

In the PIB model, the coefficient **B** is expressed as …

$$B = \left[B_{atm} + S \right]$$

where …

B_{atm} = reduction fraction per year due to natural drag (**B**<0 and **B** is inversely proportional to $\mathbf{D_1}$ – the average population object diameter)

S = reduction fraction per year due to the use of a “debris sweeper” (**S**<0 and may take on any assigned value – typically -0.005 to -0.05)

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The PIB Model – C: The Collision Coefficient

In the PIB model, the coefficient **C** is expressed as the product of two terms …

$$C = (\delta)\dot{H}_{11}$$

where …

δ = the number of fragments produced per collision minus the two destroyed during a collision

$\dot{H}_{11}$ = the collision frequency between members of a population of identical objects (This term is presented explicitly in the next slide.)

The PIB Model – Collision Frequency Between Identical Objects

$$\dot{H}_{11} = (F_{\nu})\left\{\frac{(\sqrt{2}V_c)D_1^{\,2}}{(4/3)(R_T^3 - R_B^3)}\right\}\left(\frac{1-1/N_1}{2}\right)$$

where …

Fν = “incomplete mixing factor” – acknowledges that not all objects have access to all parts of the PIB “box”

V_c = orbital speed at the average population altitude

D_1 = the average population object diameter

N_1 = the number of objects having diameter **D_1**

R_T = radius – top of the LEO environment shell from Earth’s center

R_B = radius – bottom of the LEO environment shell from Earth’s center

The PIB Model – Regarding LEO Environmental Stability

The form of the PIB environment evolution equation is that of a simple quadratic equation. During the computational execution of the PIB model all relevant parameters are evaluated at the end of each time step including the coefficients **A**, **B**, and **C** as well as the average particle characteristics. Therefore, it is possible to solve for the roots of the PIB equation by application of the quadratic formula shown here as . . .

$$\dot{H}_{11} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2C}$$

(Note that this author has interchanged the roles of **A** and **C** with respect to the usual presentation of the quadratic equation.)

The PIB Model – Regarding LEO Environmental Stability

The quantity under the radical is identified as …

$$q = B^2 - 4AC$$

These terms may be described qualitatively as …

$$q = [SINKS] - [SOURCES]$$

The quantity "q" involves the difference between the **A** and **C** source terms and the **B** sink term. Three types of behavior may be identified …

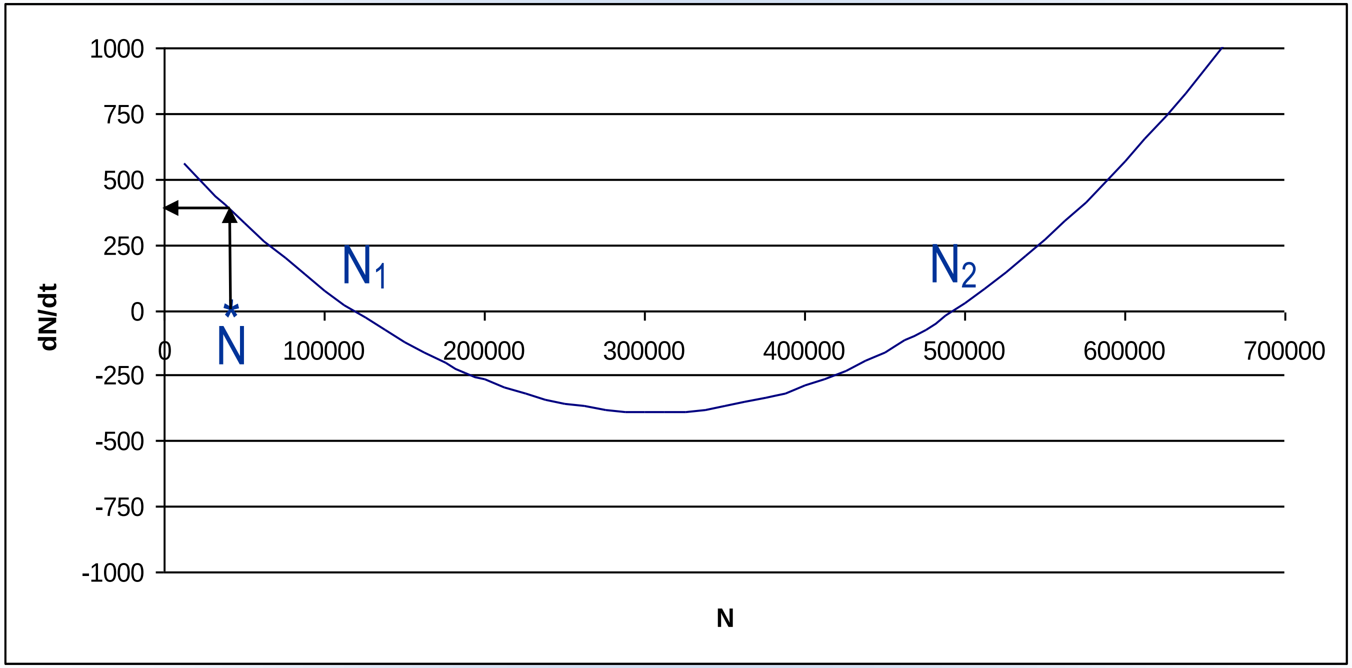
$q > 0 : [SINKS] > [SOURCES]$: **Conditionally Stable**

$q = 0 : [SINKS] = [SOURCES]$: **Instability Threshold**

$q < 0 : [SINKS] < [SOURCES]$: **Unconditionally Unstable**

The PIB Model – Regarding LEO Environmental Stability

Consider the illustrative case below for which N_1 = 120,000 and N_2 = 490,000 …



As described in this example of "conditionally stable" behavior, the number of objects in LEO, **N**, always approaches the equilibrium value $\mathbf{N_1}$, as long as it is not suddenly perturbed – by accidental or deliberate action – to a state characterized by a value of **N** > $\mathbf{N_2}$ for which d**N**/dt > 0 for all **N**.

The PIB Model – Long Term LEO Evolution Illustrated – No Large Object Removal

Using "nominal" parameters, the PIB model predicts …

(1) The LEO environment is currently unstable and will not exhibit two real roots until 2400.

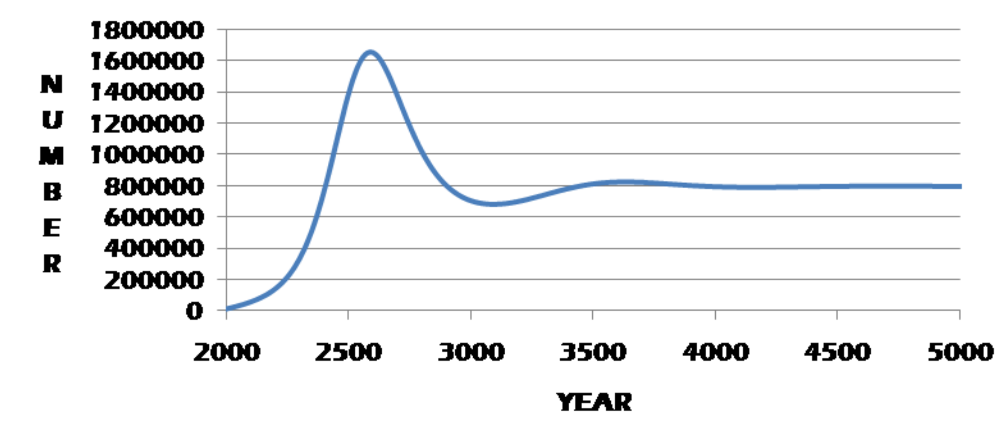
(2) LEO debris growth may be expected to continue until 2590 because the total population, **N**, will continue to be greater than $\mathbf{N_2}$.

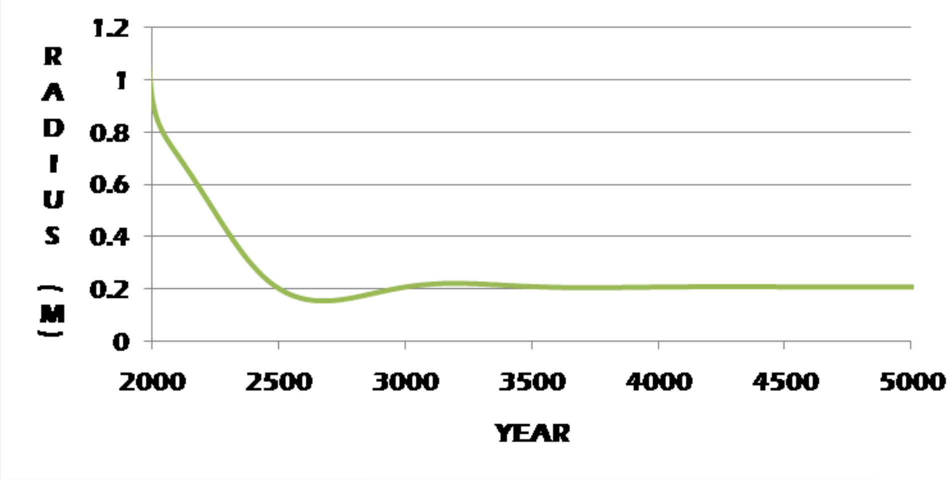
(3) Throughout the growth period, the average object size will get smaller due primarily to collisions.

(4) As the average particle size diminishes, the efficacy of atmospheric drag increases -- $\mathbf{N_1}$ will become smaller and $\mathbf{N_2}$ will become larger -- eventually overtaking **N** in 2590.

(5) The population reaches a peak of 1,653,000 in 2590 and then begins to decay.

(6) Eventually, a stable environment is achieved -- determined by the asymptotic steady-state value of $\mathbf{N_2} = \mathbf{N}$ = 795,000.





"Target Area" in LEO – NOV. 2009

DATA SOURCE: NOV2009 RCS data source =**CelesTrak (http://celestrak.com/).**
OBJECTS OF INTEREST: Resident and/or transient in the altitude regime to 2000 km.
NUMBER OF OBJECTS IDENTIFIED: 12619 with reported RCS values.
SUM TOTAL RCS OF THIS POPULATION: 16122.244 m^2
AVERAGE CHARACTERISTICS PER PIECE: <area> = 1.2776 m^2 / <D> = 1.2754 m.

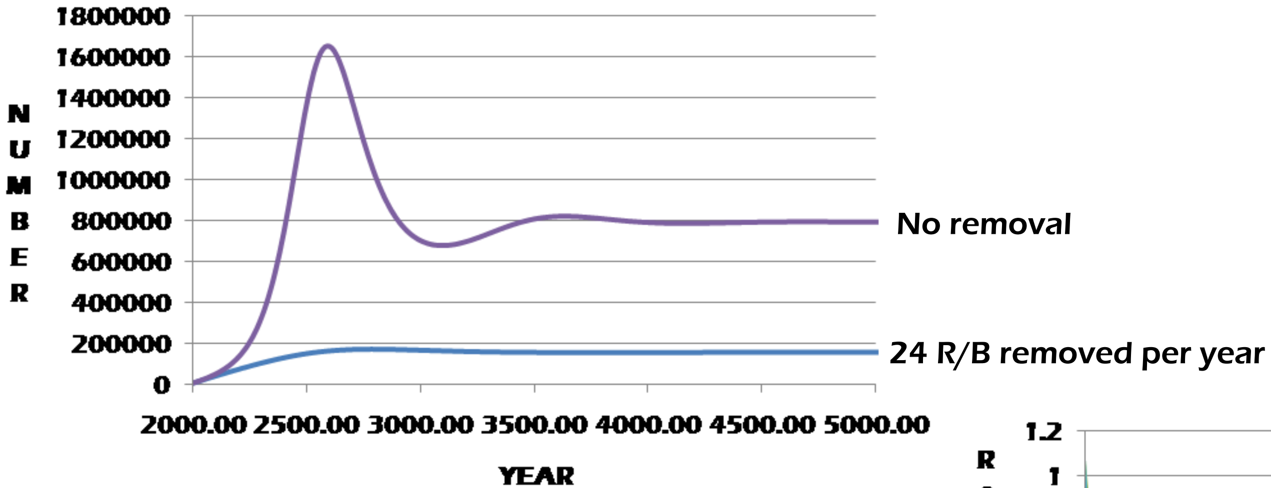
STATISTICS – WHERE IS THE COLLISIONAL CROSS-SECTION FOUND?

N Largest Objects	% of LEO Population	% of "Target" Area
39	0.31%	10.00%
126	1.00%	22.00%
630	5.00%	58.21%
1262	10.00%	78.59%

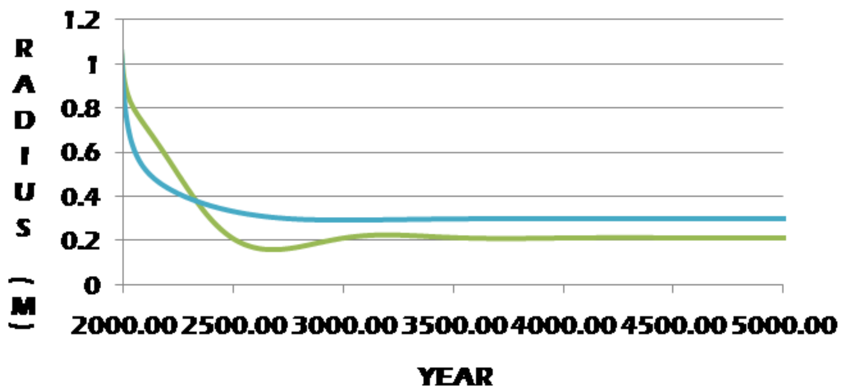
Large Object Cross-Sections – Specific Object Types

Object Name	#Pieces	Cum. Area (m^2)	<Area >	<Dia.>
ARIANE 2 R/B	4	26.067	6.517	2.881
ARIANE 3 R/B	4	63.95	15.987	4.512
ARIANE 44L R/B	18	420.587	23.366	5.454
ARIANE 5 R/B	37	896.174	24.221	5.553
ATLAS 2 CENTAUR	5	80.37	16.074	4.524
ATLAS 2A CENT.	9	142.477	15.831	4.49
ATLAS CENT. R/B	25	371.392	14.856	4.349
R/B Averages -->	102	2001.017	19.6178	4.998
ARIANE 1 DEB	69	24.535	0.356	0.673
ARIANE 2 DEB	85	57.545	0.677	0.928
ARIANE 3 DEB	25	73.996	2.96	1.941
CBERS 1 DEB	75	1.852	0.025	0.178
COS. 1275 DEB	256	17.091	0.067	0.292
COS. 1375 DEB	57	4.12	0.072	0.303
COS. 2251 DEB	1062	27.887	0.026	0.182
FENGYUN 1C DEB	2631	71.173	0.027	0.185
Small Debris Ave.-->	4260	278.199	0.0653	0.288

The PIB Model – Long Term LEO Evolution Illustrated – With Large Object Removal



The removal of two 20 sq-meter rocket bodies (or equivalent) per month can mean the difference between "catastrophic runaway" or relatively benign growth. The next task is then to develop a prioritization list with the objective of removing those objects that are the greatest threat from a future collisions perspective.



Building on The PIB Concept – Multiple Particle Types

The development of a multi-particle-type model from the modeling protocol of the original PIB model was described in Talent (1992) and based on the following mathematical prescriptions. The basic PIB, extended to the case of a single environmental box containing *m* species, becomes …

$$\dot{N}_k = A_k + B_k N_k + \sum_{i=1}^{k} \sum_{j=1}^{m} \delta_{(ij)k} \dot{H}_{ij} N_i N_j$$

… where the index *k* may take on values from 1 to *m*. Regarding the $\dot{H}_{ij}$ factor, the earlier equation from the PIB is appropriate if $i = j$; for dissimilar objects the appropriate form is …

$$\dot{H}_{ij} = (F_{Vij}) \left\{ \frac{(\sqrt{2} V_c)\left[(D_i + D_j)/2\right]^2}{(4/3)(R_T^3 - R_B^3)} \right\}$$

All notation is similar to the definitions used for the basic PIB model. Perhaps the only additional explanation necessary is to note that $\delta_{(ij)k}$ means "the number of k-type objects produced during an i-j collision."

Developing A Debris Removal Prioritization Methodology

$$\dot{H}_{ij} = (F_{vij})\left\{\frac{(V_{rel})\left[(D_i + D_j)/2\right]^2}{(4/3)(R_{Ti}^3 - R_{Bi}^3)}\right\}$$

ASSERTION: Utilizing elset data and size data (from multiple sources), one may employ the above expression to assess the magnitude of the overall threat posed by each object to the rest of the "targets" in the environment.

PROCEDURAL STEPS:

- Let object "1" be the largest member of the population – the calculation of $H_{1,2}$ may be performed paying particular attention to the calculation of (F_{vij}).
- Repeat for each member of the population – $H_{1,3}$, $H_{1,4}$, $H_{1,5}$, $H_{1,6}$, $H_{1,7}$, … $H_{1,13000}$.
- Sum over the population to assess the risk associated with this object .
- Repeat for every object in the population.
- Each object may then be ranked in order of relative threat to the environment.
- Thus, the information generated by this analysis will allow for the selection of debris targets for removal achieving maximum benefit per unit cost.

Example Output

NAME	NUMBER	APOGEE (KM)	PERIGEE (KM)	RCS (M^2)	COLL. RATE (YR^-1)	MEAN TIME BET. COLL.
METEOR-M	35865	820.00	818.00	80.2080000	0.000985198300	1015.02
IRIDIUM 86	25528	780.00	776.00	11.2670000	0.000480267900	2082.17
IRIDIUM 65	25288	780.00	776.00	9.8670000	0.000433509300	2306.76
COSMOS 1844	17973	870.00	824.00	24.2130000	0.000429597000	2327.76
RADARSAT 2	32382	792.00	791.00	22.9427000	0.000394725200	2533.41
SL-12 R/B(2)	15334	848.00	840.00	20.2287000	0.000389839300	2565.16
COSMOS 2322	23704	856.00	841.00	17.1663000	0.000359073300	2784.95
IRIDIUM 5	24795	780.00	776.00	7.2110000	0.000343076400	2914.80
OAO 1	2142	786.00	775.00	11.8790000	0.000334824300	2986.64
COSMOS 2278	23087	854.00	842.00	16.3025000	0.000333781700	2995.97
IRIDIUM 77	25471	780.00	775.00	8.1150000	0.000329500700	3034.90
IRIDIUM 6	24794	779.00	776.00	11.0501000	0.000328961700	3039.87
IRIDIUM 50	25172	779.00	776.00	10.8260000	0.000323406000	3092.09
IRIDIUM 91	27372	780.00	775.00	7.8901000	0.000322478300	3100.98
ERS 2	23560	785.00	783.00	15.4747000	0.000321155600	3113.76
IRIDIUM 37	24968	779.00	776.00	10.6890000	0.000320006300	3124.94
ARIANE 5 R/B	27387	797.00	751.00	16.6964000	0.000309304400	3233.06
ADEOS	24277	797.00	796.00	18.1484000	0.000307118600	3256.07

The Top 24 R/BS to Remove

NAME	NUMBER	APOGEE (KM)	PERIGEE (KM)	RCS (M^2)	COLL. RATE (YR^-1)	MEAN TIME BET. COLL.
SL-12 R/B(2)	15334	848.00	840.00	20.2287000	0.000389839300	2565.16
ARIANE 5 R/B	27387	797.00	751.00	16.6964000	0.000309304400	3233.06
SL-12 R/B(2)	15772	848.00	798.00	17.3215000	0.000304558800	3283.44
ARIANE 40 R/B	25261	789.00	783.00	13.9440000	0.000299131700	3343.01
SL-16 R/B	26070	855.00	828.00	13.3889000	0.000264965800	3774.07
SL-16 R/B	20625	854.00	835.00	12.0730000	0.000251237400	3980.30
SL-16 R/B	25407	846.00	836.00	12.3450000	0.000241528300	4140.30
ARIANE 40 R/B	20443	779.00	764.00	8.2350000	0.000237426200	4211.84
SL-16 R/B	23705	853.00	833.00	11.3163000	0.000233493400	4282.78
SL-16 R/B	22220	849.00	828.00	11.8810000	0.000231972600	4310.85
ARIANE 40 R/B	21610	764.00	759.00	12.8633000	0.000227212800	4401.16
SL-16 R/B	25400	815.00	801.00	15.1320000	0.000227137900	4402.61
SL-16 R/B	16182	845.00	833.00	11.6200000	0.000225300500	4438.52
ARIANE 40+ R/B	23561	775.00	766.00	8.5290000	0.000223981600	4464.65
SL-16 R/B	24298	861.00	841.00	9.6595000	0.000216815400	4612.22
SL-16 R/B	22566	850.00	837.00	9.8988000	0.000212042500	4716.04
DELTA 2 R/B	27665	805.00	793.00	10.7222000	0.000209760500	4767.34
SL-3 R/B	8027	909.00	830.00	13.1045000	0.000207997300	4807.75
H-2A R/B	27601	841.00	737.00	13.6110000	0.000205847900	4857.96
SL-16 R/B	25861	652.00	628.00	24.5580000	0.000200671600	4983.27
ATLAS CENTAUR R/B	3598	754.00	681.00	17.9908000	0.000196869800	5079.50
SL-16 R/B	23088	847.00	842.00	9.0410000	0.000196322700	5093.66
SL-3 R/B	7575	893.00	822.00	11.1950000	0.000189674200	5272.20
SL-8 R/B	24955	982.00	936.00	12.5257000	0.000188271500	5311.48
SUMMRY DATA--->	**24**	**829.63**	**800.08**	**317.8806000**	**0.005691364100**	**175.70**

Conclusions

(1) The simple PIB model shows there is a threat of catastrophic runaway growth in the orbital debris environment that would not resolve itself for hundreds of years.

(2) Although individual small debris objects are currently the most significant threat to individual high-value assets, the primary threat to the orbital debris environment in the long term may be argued to be the repository of collective collisional cross-sectional area resident in large derelict objects, such as rocket bodies, since they will be the primary source of small debris in the future.

(3) Removing as few as two large derelicts per month can make a significant difference in the future state of the LEO environment.

(4) The collision rate equation can be used as a basis for prioritizing the removal of large objects. A successful demonstration of that has been performed here, suggesting that the production of a strategic decision analysis tool may indeed be worth while.

Dedication

Finally, I wish to dedicate this work to my twin sons – Spc. Aaron G. Talent (U.S. Army) and Sgt. Byron K. Talent (U.S. Army) – recent veterans of Operation Iraqi Freedom.

Aaron

Byron

References

Kessler, D.J., and Cour-Palais, B. G., "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," *Journal of Geophysical Research,* Vol. 83, No. A6, pp. 2637-2646, 1978.

Talent, D.L., "Analytical Model for Orbital Debris Environment Management," *AIAA/NASA/DoD Orbital Debris Conference: Technical Issues & Future Directions*, Paper # AIAA 90-1363, 1990.

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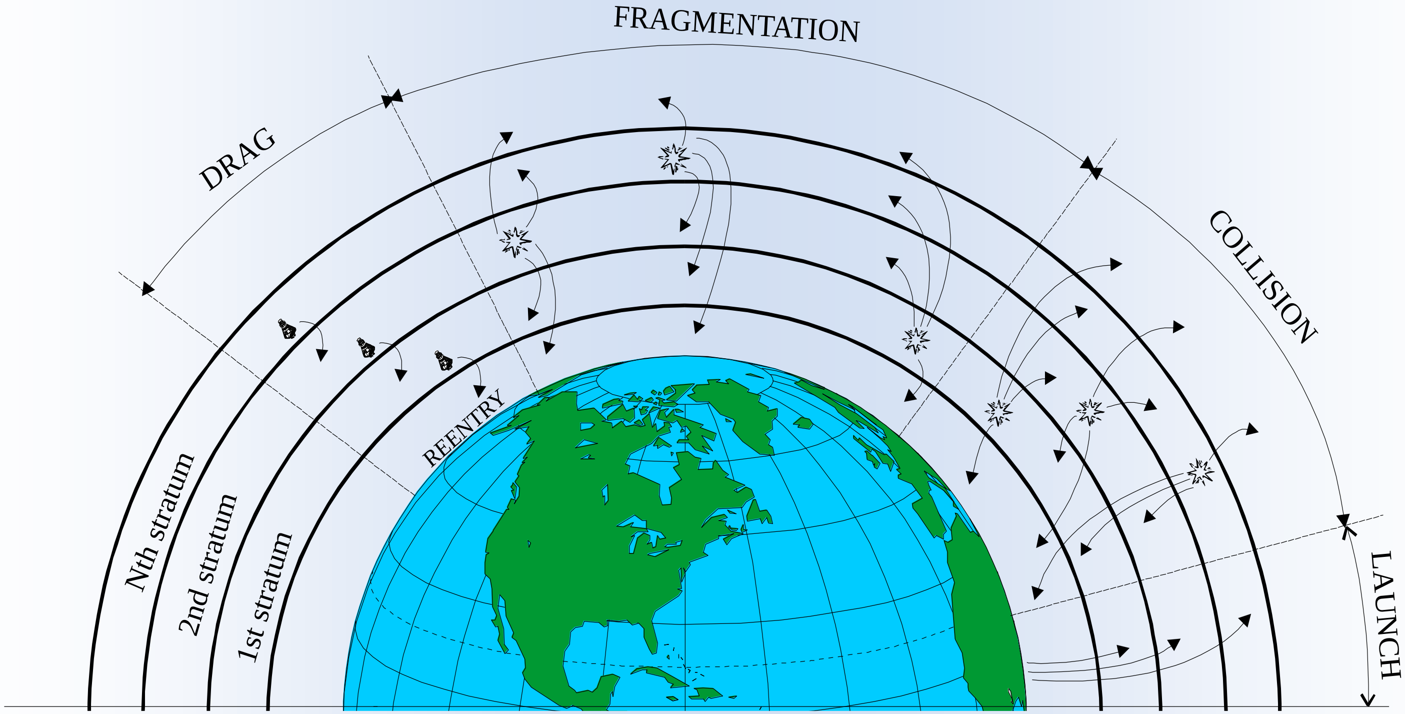
BACKUP SLIDES

Building on The PIB Concept – Multiple Particle Types / Multiple Strata

To extend the PIB model to a *n*-multistrata system of *m-species* , *(nxm)* population evolution equations, $\dot{N}_k$, are written. These equations will also include crossfeed terms to accommodate migration from stratum to stratum and/or multistrata/multisize deposition due to fragmentations and collisions. These interactions are illustrated below.



Building on The PIB Concept – Definition of the Multiple Strata

In the current version of the PODEM model, ten LEO environmental strata are defined as follows …

S1, 350-410 km	S6, 690-780 km
S2, 410-470 km	S7, 780-925 km
S3, 470-535 km	S8, 925-1180 km
S4, 535-605 km	S9, 1180-1550 km
S5, 605-690 km	S10, 1550-2000 km

Five discrete object types are defined in PODEM, characterized here by their drag coefficients, C_d(A/M). These values are, specifically …

T1, 0.001394; T2, 0.004915; T3, 0.01843; T4, 0.0360, and; T5, 0.1234

Note:
T1 and T2 objects -- representative of intact satellites and rocket bodies.
T3 and T4 objects -- representative of fragmentation and collision debris large enough to catalog.
T5 objects – representative of the small, untracked component of the LEO population.

The PIB Model – "Nominal Parameters"

PARAMETERS OF THE MODEL

L: LAUNCHES PER YEAR =	70.0000
P1: PIECES PER LAUNCH =	4.1100
D1: FRAC. OF PCS. SURVIVING 1 YR. =	0.6320
FE: FRAC. OF LAUNCHES PROD. EXPL. =	0.0280
DE: FRAC. OF EXPL. ABOVE 350 KM =	0.8200
PE: PIECES PRODUCED PER EXPLOSION =	125.0000
PC: PIECES PRODUCED PER COLLISION =	200.0000
VMIX: FRAC. AVAIL. FOR COLLISION =	0.5500
REM: OBJECTS RETRIEVED PER YEAR =	0.0000
ZHI: TOP OF LEO (KM) =	8378.1348
ZLO: BOTTOM OF LEO (KM) =	6728.1348
VCAVE: ORBITAL SPEED (KM/SEC) =	7.3220
RETEXP: EXPLOSION AREA INC. FACTOR =	6.0000
RETCOL: COLLISION AREA INC. FACTOR =	16.0000
STEP: FRAC. OF YEAR FOR ITER. STEP =	0.0500
ASEL: FREQ., IN YEARS, OF OUTPUT =	10.0000
ZYRS: TOTAL YRS. TO EVOLVE =	8000.0000